Resonant Frequencies of Dielectric Resonators Containing Guided Complex Modes

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***Abstract* — It is shown that unless complex modes are included in the mode matching analysis to determine the resonant frequencies of dielectric-loaded resonators, some resonant frequencies could be missed. Field distributions, mode charts, and mode coefficients of dielectric-loaded resonators in which complex modes exist are presented. Experimental measurements for the verification of the computed results are presented and show good agreement with theory.**

I. Introduction

Methods described in the literature for the determination of the resonant frequencies of dielectric-loaded resonators based on mode matching techniques [1], [2] assume that all the modes in the expansion are either propagating modes or evanescent modes of the appropriate region of the structure. It is known that complex modes [3]–[5] can exist in dielectric-loaded waveguides. Recently methods for the systematic determination of the complex modes in dielectric-loaded waveguides have been developed [6]. While investigating the optimum dimensions of dielectric resonators to produce the largest possible mode separation for filter applications [7], a resonant mode was experimentally detected which was not found by computer simulation using the mode matching method without including the complex modes. When complex modes were included in the simulation, all measured resonant frequencies were precisely predicted. The objective of this paper is to present the results of including the complex modes in the analysis of dielectric-loaded resonators.

II. Resonant Frequency Calculations and Measurements

The structure under consideration is shown in Fig. 1. It consists of a cylindrical dielectric resonator of length l, radius a, and relative dielectric constant ϵ_{r_1} coaxially and symmetrically located inside a perfectly conducting cylindrical enclosure of length L and radius b. The method of calculating the resonant frequencies of this structure for hybrid modes is to expand the transverse fields in regions A, B, and C in Fig. 1 in terms of waveguide modes that could exist in each of these regions. The boundary conditions at the interfaces are then applied and weighted in terms of the same modes. This process results in an infinite set of linear homogeneous equations in the unknown expansion coefficients. Resonant frequencies of the structure are the roots of the equation obtained from equating the determinant of the truncated matrix to zero. When complex modes are included the same procedure applies with slight modification, summarized below.

Complex modes always occur in conjugate pairs whose propagation constants are γ and γ^*. Let the transverse electric and magnetic field components of such modes be given by

$$\hat{e}_t = \hat{e}_1 + j\hat{e}_2 \qquad \hat{h}_t = \hat{h}_1 + j\hat{h}_2$$

$$\hat{e}_t^* = \hat{e}_1 - j\hat{e}_2 \qquad -\hat{h}_t^* = -\hat{h}_1 + j\hat{h}_2$$

where $\hat{e}_1$, $\hat{e}_2$, $\hat{h}_1$, and $\hat{h}_2$ are real vector functions of r. In an infinite dielectric loaded waveguide, only two of the complex modes can be excited for $z > 0$. The total field of these two modes will be

$$\boldsymbol{E} = \hat{e}_t e^{\gamma z} + \hat{e}_t^* e^{-\gamma^* z} = (\hat{e}_1 + j\hat{e}_2) e^{-\alpha z} e^{-j\beta z} + (\hat{e}_1 - j\hat{e}_2) e^{-\alpha z} e^{j\beta z}$$
$$= 2e^{-\alpha z}(\hat{e}_1 \cos\beta z + \hat{e}_2 \sin\beta z).$$

Similarly,

$$\boldsymbol{H} = 2je^{-\alpha z}(-\hat{h}_1 \sin\beta z + \hat{h}_2 \cos\beta z)$$

where $\gamma = \alpha + j\beta$. Similar expressions hold for the complex mode pair excited in $z < 0$. A computed and measured mode chart showing the variation of resonant frequency with (L/l) for a dielectric-loaded resonator with parameters: $a = 0.4$ in., $b = 0.6$ in., $l = 0.18$ in., and $\epsilon_{r_1} = 35.59$ is shown in Fig. 2. The measured data are indicated by a star in this figure. Before inclusion of the complex modes in the computer simulation, the dotted portion of the curve in Fig. 2 for the HEH_{21} mode could not be obtained at all, while all the other solid curves were actually generated. Investigation into the reasons why these resonances were not predicted started by searching for complex modes in the dielectric-loaded waveguide with the same cross-sectional dimensions as the resonator. An $(\omega - \beta)$ diagram for such a waveguide is shown in Fig. 3(a). The expanded frequency range where the relevant complex modes occur is shown in Fig. 3(b). It is seen from this figure that complex modes exist in the frequency range 4.9 GHz $< f <$ 5.56 GHz. Complex modes were then included in the computer program for the determination of the resonant frequencies and mode charts. This resulted in the prediction of the dotted curve shown in Fig. 2, which accounted for all measured resonances that were not possible to predict without inclusion of the complex modes.

Variation of the normalized values of the squares of the expansion coefficients as a function of the ratio (L/l) is shown in Fig. 4(a). The same coefficients are shown on an expanded frequency scale in Fig. 4(b). In this figure complex modes exist in the region $(L/l) > 1.79$. Just before the appearance of the complex modes the two propagating modes (HE_{23} and HE_{24}) become the dominating modes in the resonator, with each accounting for approximately 50 percent of the total stored energy in the resonator. Almost no other modes exist in the resonator at this value of (L/l).

Field intensities and distributions in the resonator with complex mode contents were also calculated. Fig. 5(a)–(d) shows field plots for the HEH_{21} mode with the complex modes included. These fields do not differ substantially in their general behavior from the cases presented in [1], where the complex modes were not excited.

III. Conclusion

Whenever complex modes are excited in any region of a dielectric-loaded resonator, they must be included in the field expansions using the mode matching technique in order to predict all the possible resonances. These modes appear in complex conjugate pairs and occur in a limited range of structure parameters and frequency bands. Field distributions in dielectric-loaded resonators where complex modes exist are similar in nature to the cases where the complex modes are not excited. In some cases complex modes could account for almost all the stored energy in a dielectric-loaded resonator.

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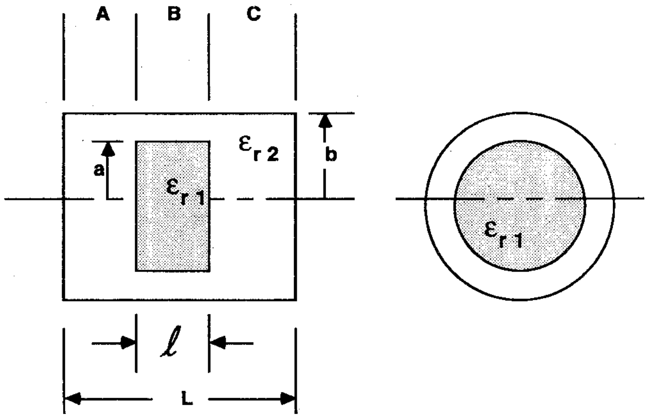

Fig. 1. Dielectric-loaded resonator.

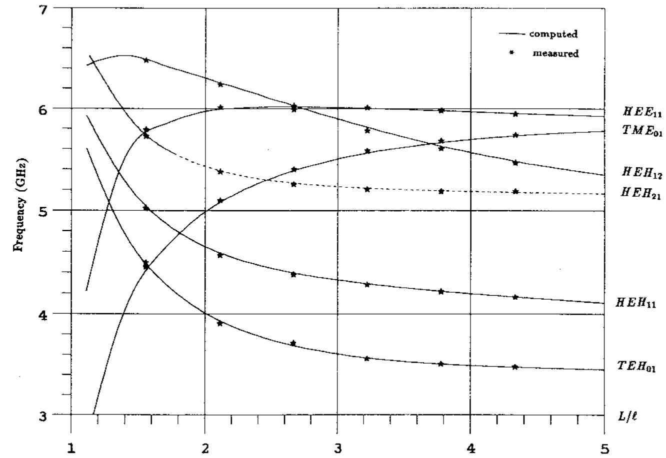

Fig. 2. Mode chart of dielectric-loaded resonator.

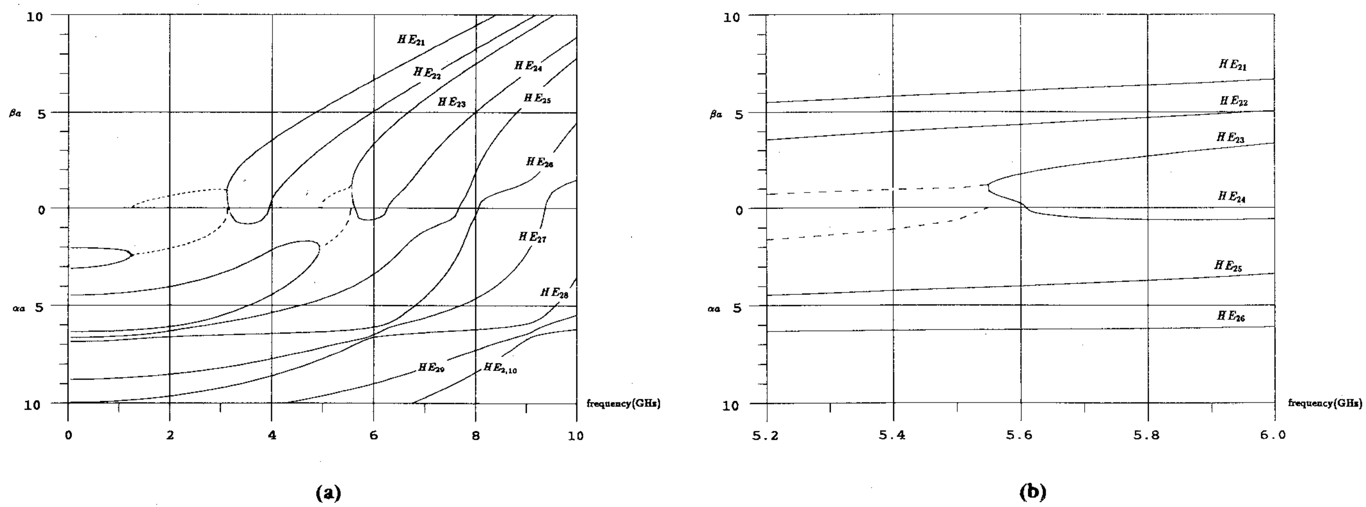

Fig. 3. (a) $\omega-\beta$ diagram of a dielectric-loaded waveguide with the same cross section as the dielectric resonator. (b) Enlarged $\omega-\beta$ diagram.

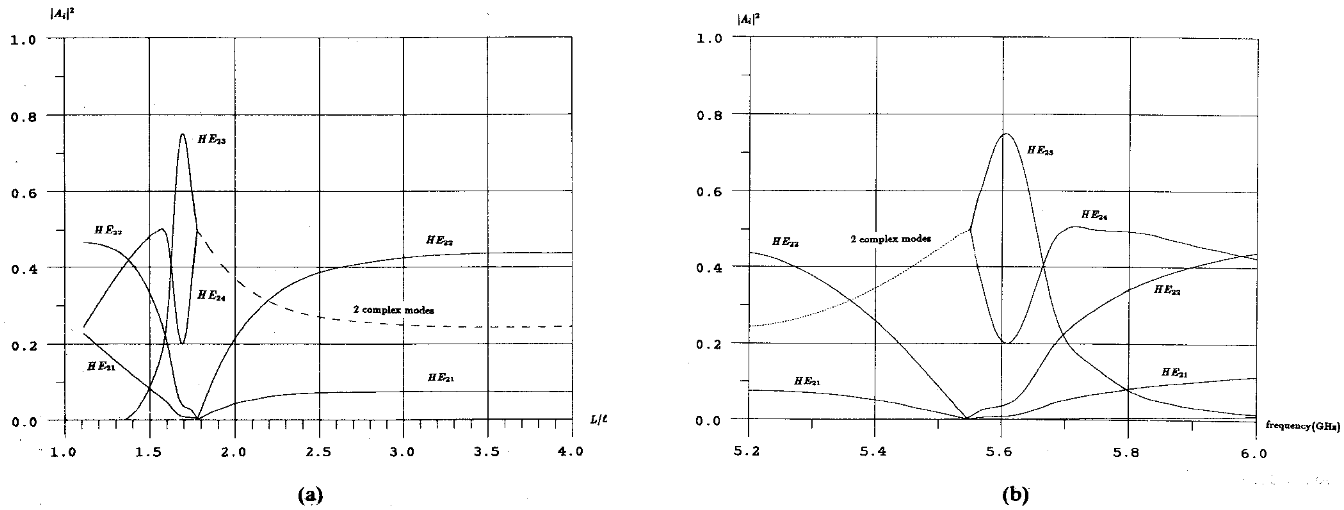

Fig. 4. Variation of the mode expansion coefficients in the dielectric-loaded region with L/l. (b) Variation of mode expansion coefficients with frequency.

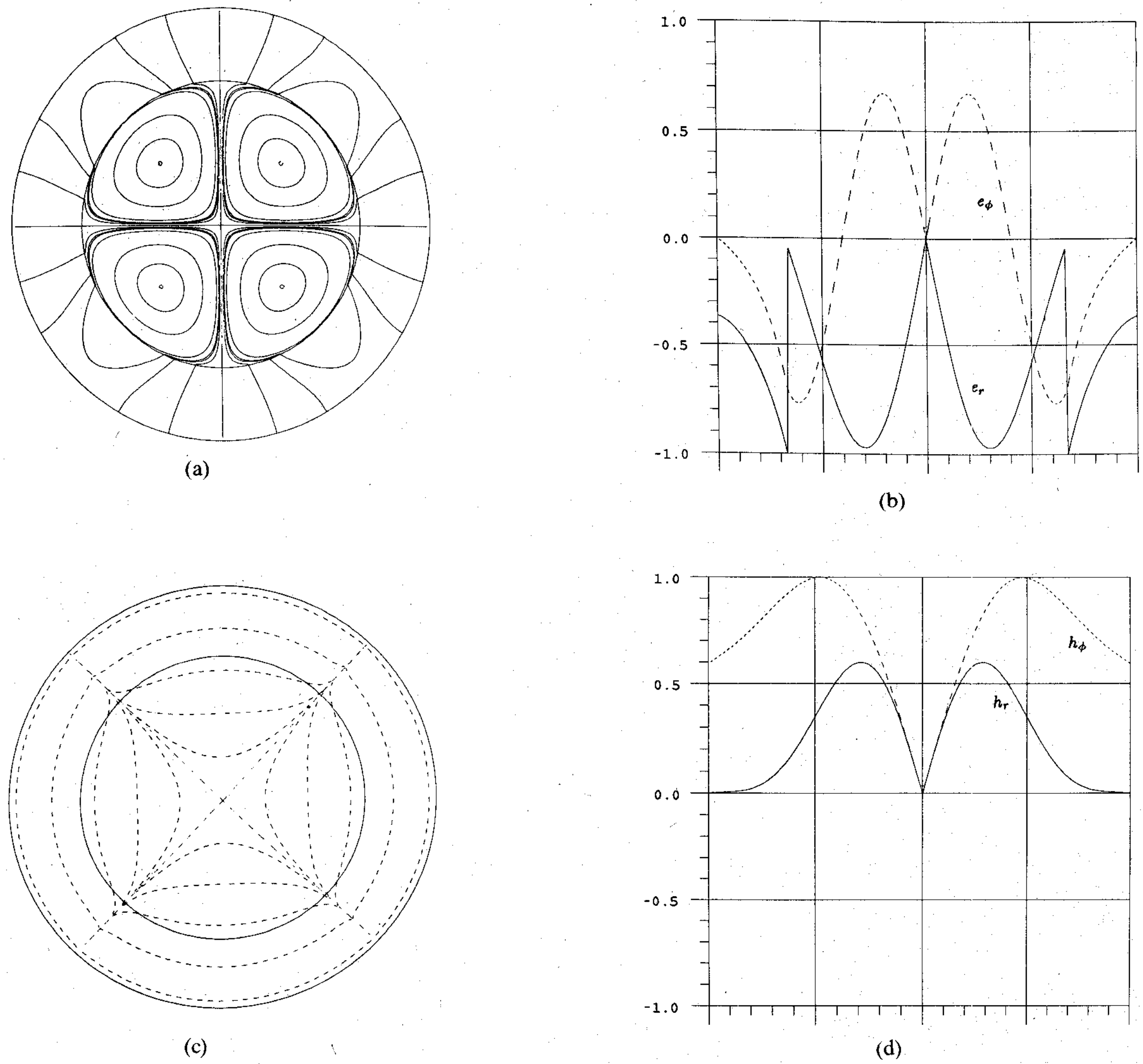

Fig. 5. (a) Electric field distribution of HEH_{21} at $z = 0$ (complex modes are included in mode expansion). (b) Electric field intensity variation of HEH_{21} at $z = 0$. (c) Magnetic field distribution of HEH_{21} at $z = L/l$ (complex modes are included in mode expansion). (d) Magnetic field intensity variation of HEH_{21} at $z = L/l$.

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